MULTIANTENNA DETECTION UNDER NOISE UNCERTAINTY AND PRIMARY USER'S SPATIAL STRUCTURE

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ABSTRACT

Spectrum sensing is a challenging key component of the Cognitive Radio paradigm, since primary signals must be detected in the face of noise uncertainty and at signal-to-noise ratios (SNRs) well below decodability levels. Multiantenna detectors exploit spatial independence of receiver thermal noise to boost detection performance and robustness. Here, we study the problem of detecting Gaussian signals with unknown rank-P spatial covariance matrix when the noise at the receiver is independent across the antennas and with unknown power. A generic diagonal noise covariance matrix is allowed to model calibration uncertainties in the different antenna frontends. We derive the generalized likelihood ratio test (GLRT) for this detection problem. Although in general the corresponding statistic must be obtained by numerical means, in the low SNR regime the GLRT does admit a closed form. Numerical simulations show that the proposed asymptotic detector offers a good performance even for moderate SNR values.

Index Terms— Cognitive radio (CR), spectrum sensing, generalized likelihood ratio test (GLRT), maximum likelihood (ML) estimation

1. INTRODUCTION

The Cognitive Radio (CR) paradigm aims to improve wireless spectrum usage and alleviate the apparent scarcity of spectral resources as seen today [1,2]. The key idea behind CR is to allow opportunistic access to temporally and/or geographically unused licensed bands, avoiding conflicts with the rightful license owners (primary users) in those bands. Thus, CR necessarily relies on powerful spectrum sensing algorithms to identify spectrum holes. However, sensing the wireless medium is a very challenging task due to fading and shadowing phenomena, which may result in very low signal-to-noise ratio (SNR) operation conditions [3].

In principle, detectors may exploit certain features of the primary signal such as the presence of pilots and/or cyclostationarity. However, most of such approaches assume some level of synchronization with the primary signal, and at very low SNRs the synchronization loops of the monitoring system cannot be expected to provide the required accuracy for the carrier frequency and/or clock rate estimates [3]. These issues are avoided by detectors whose operation does not require synchronization. The most popular of these asynchronous detectors is the energy detector, which does not exploit any *a priori* knowledge about the signal structure. However, this detector requires knowledge of the noise variance to compute the decision threshold, and any uncertainty regarding this parameter translates in severe performance degradation [4]. Multiple-antenna detectors are a promising approach to alleviate the noise uncertainty problem. This idea has been explored by different authors [5–7] under the following assumptions: (i) temporally white Gaussian model for both signal and noise, (ii) spatially white noise with the same unknown variance across antennas, and (iii) an unknown rank-1 spatial covariance matrix for the signal.

In this paper we consider the case in which the received signals present a spatial rank $P \geq 1$. This might happen, for example, if several independent primary users (e.g. from adjacent cells) simultaneously access the same frequency channel, or if the primary transmitter uses multiple antennas to achieve multiplexing gain and/or enhance spatial diversity. Additionally, we allow a generic diagonal noise covariance to model calibration uncertainties in the different antenna frontends. Under this model we derive the generalized likelihood ratio test (GLRT), which results in an optimization problem with no closed-form solution in the general case. We show that for asymptotically low SNR the GLRT can be written in closed form depending only on the P largest eigenvalues of the sample coherence matrix. The proposed detector generalizes the detectors derived in [8] and [9] for $P = 1$ and large P, respectively.

2. PROBLEM FORMULATION

Let us consider a cognitive radio node equipped with L antennas which senses a given frequency channel. The received signals are downconverted and sampled at the Nyquist rate assuming no synchronization with any potentially present primary signal. Assuming a frequency-flat channel, the hypothesis testing problem can be written as

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{v}, \\ \mathcal{H}_0 &: \mathbf{x} = \mathbf{v}, \end{aligned} \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^P$ is the temporally white primary signal, $\mathbf{H} \in \mathbb{C}^{L \times P}$ is the unknown multiple-input multiple-output (MIMO) channel between the primary user and the spectrum monitor, and $\mathbf{v} \in \mathbb{C}^L$ is the additive noise, which is assumed to be zero-mean circular complex Gaussian, spatially uncorrelated and temporally white.

Before proceeding, we need the probability density function of $\mathbf{s}$. We consider a zero-mean circular complex Gaussian model, which is particularly accurate if the primary transmitter uses orthogonal frequency division multiplexing (OFDM). Even if this is not the case, the Gaussian model leads to tractable analysis and useful detectors. We may assume that $\mathbf{s}$ is spatially white with unit-power components, as any spatial correlation and scaling of the primary signal can be absorbed in the channel matrix $\mathbf{H}$ without altering the statistical model. Under these assumptions, the spatial covariance matrices of the primary signal and noise are given by

$$E\left[\mathbf{s}\mathbf{s}^H\right] = \mathbf{I}_P, \qquad E\left[\mathbf{v}\mathbf{v}^H\right] = \mathbf{\Sigma}^2, \tag{2}$$

where $\mathbf{\Sigma}^2$ is an unknown diagonal covariance matrix with positive entries and $\mathbf{I}_P$ is the identity matrix of dimensions $P \times P$. Hence, the detection problem in (1) is a test for the covariance structure of the vector-valued random variable $\mathbf{x}$, i.e.,

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}_L, \mathbf{H}\mathbf{H}^H + \mathbf{\Sigma}^2\right), \\ \mathcal{H}_0 &: \mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}_L, \mathbf{\Sigma}^2\right), \end{aligned} \tag{3}$$

where $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{R})$ stands for the complex circular Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{R}$. Therefore, under $\mathcal{H}_0$ the covariance matrix $\mathbf{R}$ is diagonal, whereas under $\mathcal{H}_1$ it is a rank-P matrix plus a diagonal term. We shall assume that $\mathbf{H}$ has full column rank.

3. DERIVATION OF THE GLRT

In this section we consider the detection problem given in (3). As there are unknown parameters under both hypotheses, this is a composite test and the Neyman-Pearson detector is not implementable. We apply instead the generalized likelihood ratio test (GLRT), since it usually results in simple detectors with good performance [10]. We shall consider $M \geq L$ snapshots $\mathbf{x}_0, \ldots, \mathbf{x}_{M-1}$. Assuming a block-fading channel, that is, the channel remains approximately constant during the sensing time window, these can be regarded as iid realizations of $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}_L, \mathbf{R})$. Hence, the likelihood is given by the product of the individual pdfs, i.e.,

$$p\left(\mathbf{x}_0, \ldots, \mathbf{x}_{M-1}; \mathbf{R}\right) = \frac{1}{\pi^{LM} \det\left(\mathbf{R}\right)^M} \exp\left\{-M \operatorname{tr}\left(\hat{\mathbf{R}}\mathbf{R}^{-1}\right)\right\}, \tag{4}$$

where $\hat{\mathbf{R}} = \frac{1}{M}\sum_{m=0}^{M-1} \mathbf{x}_m \mathbf{x}_m^H$ is the sample covariance matrix. The GLRT for $\mathcal{H}_0 : \mathbf{R} = \mathbf{\Sigma}^2$ vs. $\mathcal{H}_1 : \mathbf{R} = \mathbf{H}\mathbf{H}^H + \mathbf{\Sigma}^2$ is based on the generalized likelihood ratio $\mathscr{L}$ [10]

$$\mathscr{L} = \frac{\max_{\mathbf{\Sigma}^2} p\left(\mathbf{x}_0, \ldots, \mathbf{x}_{M-1}; \mathbf{\Sigma}^2\right)}{\max_{\mathbf{H}, \mathbf{\Sigma}^2} p\left(\mathbf{x}_0, \ldots, \mathbf{x}_{M-1}; \mathbf{H}, \mathbf{\Sigma}^2\right)}, \tag{5}$$

which is compared against a threshold in order to decide if the primary signal is present or absent.

To derive the GLRT, we need the maximum likelihood (ML) estimates of the unknown parameters. The ML estimate of $\mathbf{\Sigma}^2$ under $\mathcal{H}_0$ can be straightforwardly obtained [9], and is given by

$$\hat{\mathbf{\Sigma}}^2 = \operatorname{diag}(\hat{\mathbf{R}}) \doteq \hat{\mathbf{D}}, \tag{6}$$

where $\operatorname{diag}(\boldsymbol{A})$ denotes a diagonal matrix with diagonal equal to that of $\boldsymbol{A}$.

Under $\mathcal{H}_1$, we need the ML estimates of $\mathbf{\Sigma}^2$ and $\mathbf{H}$. As can be expected, these estimates depend on the signal rank P. If P is sufficiently large, the model imposes no useful constraints on the covariance matrix, as the following result establishes:

Lemma 1. *If $P \geq L - \sqrt{L}$, the ML estimates of $\mathbf{H}$ and $\mathbf{\Sigma}^2$ under $\mathcal{H}_1$ satisfy $\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \hat{\mathbf{\Sigma}}^2 = \hat{\mathbf{R}}$.*

Proof: The proof can be found in [9, 11]. It hinges on the fact that if $P \geq L - \sqrt{L}$, then $\mathbf{H}\mathbf{H}^H + \mathbf{\Sigma}^2$ has no further structure beyond being positive definite Hermitian. ■

For unstructured covariance matrices, i.e. $P \geq L - \sqrt{L}$, the GLRT is given by the Hadamard ratio of the sample covariance matrix [9]:

$$\mathscr{L} = \frac{\det(\hat{\mathbf{R}})}{\prod_{i=1}^{L} [\hat{\mathbf{R}}]_{i,i}}. \tag{7}$$

On the other hand, for $P < L - \sqrt{L}$ the low-rank structure of the primary signal can be exploited in order to improve detection performance. Let us define $\hat{\mathbf{R}}_{\mathbf{\Sigma}} = \mathbf{\Sigma}^{-1}\hat{\mathbf{R}}\mathbf{\Sigma}^{-1}$ and $\mathbf{H}_{\mathbf{\Sigma}} = \mathbf{\Sigma}^{-1}\mathbf{H}$. Then, the log-likelihood can be rewritten as

$$\begin{aligned} \log p\left(\mathbf{x}_0, \ldots, \mathbf{x}_{M-1}; \mathbf{H}_{\mathbf{\Sigma}}, \mathbf{\Sigma}^2\right) &= -\log\det\left(\mathbf{\Sigma}^2\right) \\ - \log\det\left(\mathbf{H}_{\mathbf{\Sigma}}\mathbf{H}_{\mathbf{\Sigma}}^H + \mathbf{I}\right) &- \operatorname{tr}\left[\hat{\mathbf{R}}_{\mathbf{\Sigma}}\left(\mathbf{H}_{\mathbf{\Sigma}}\mathbf{H}_{\mathbf{\Sigma}}^H + \mathbf{I}\right)^{-1}\right]. \end{aligned} \tag{8}$$

Taking into account the eigenvalue decomposition (EVD) of $\mathbf{H}_{\mathbf{\Sigma}}\mathbf{H}_{\mathbf{\Sigma}}^H$, which is given by $\mathbf{H}_{\mathbf{\Sigma}}\mathbf{H}_{\mathbf{\Sigma}}^H = \mathbf{G}\mathbf{\Phi}^2\mathbf{G}^H$, the ML estimates of $\mathbf{G}$ and $\mathbf{\Phi}^2$ are given in Lemma 2.

Lemma 2. *Let* $\hat{\mathbf{R}}_{\mathbf{\Sigma}} = \mathbf{Q}\,\mathrm{diag}\,(\gamma_1,\ldots,\gamma_L)\,\mathbf{Q}^H$ *be an EVD of* $\hat{\mathbf{R}}_{\mathbf{\Sigma}}$*, with* $\gamma_1 \geq \cdots \geq \gamma_L$*. The ML estimates of* $\mathbf{G}$ *and* $\mathbf{\Phi}^2 = \mathrm{diag}\,(\phi_1,\ldots,\phi_L)$ *(which are functions of* $\mathbf{\Sigma}^2$*) are*

$$\hat{\mathbf{G}} = \mathbf{Q}, \quad (9)$$

$$\hat{\phi}_i^2 = \begin{cases} \gamma_i - 1, & i = 1,\ldots,P, \\ 0, & i = P+1,\ldots,L. \end{cases} \quad (10)$$

Proof: Once $\hat{\mathbf{R}}$ and $\mathbf{H}$ have been prewhitened, the proof follows the same lines as those in [12]. ■

Substituting the ML estimate of $\mathbf{H}_{\mathbf{\Sigma}}\mathbf{H}_{\mathbf{\Sigma}}^H$ into (8) yields

$$\log p\left(\mathbf{x}_0,\ldots,\mathbf{x}_{M-1};\mathbf{\Sigma}^2\right) = -\log\det(\hat{\mathbf{R}}) - \sum_{i=P+1}^{L}\left[\gamma_i - \log\gamma_i\right]. \quad (11)$$

Note that (11) is maximized w.r.t. $\gamma_{P+1},\ldots,\gamma_L$ when $\gamma_{P+1} = \ldots = \gamma_L = 1$. However, this point is not necessarily reachable, since the eigenvalues of $\hat{\mathbf{R}}_{\mathbf{\Sigma}} = \mathbf{\Sigma}^{-1}\hat{\mathbf{R}}\mathbf{\Sigma}^{-1}$ cannot be arbitrarily selected in general by choice of $\mathbf{\Sigma}$. To the best of our knowledge, the maximization of (11) with respect to $\mathbf{\Sigma}^2$ does not admit a closed-form solution in general. In fact this problem is not convex and may present multiple local maxima. An exception occurs if $\hat{\mathbf{R}}$ is diagonal, since in that case $\mathbf{\Sigma}^2 = \mathrm{diag}(\hat{\mathbf{R}})$ results in $\hat{\mathbf{R}}_{\mathbf{\Sigma}} = \mathbf{I}$, which is clearly optimal.

Now, for asymptotically low SNR the sample covariance matrix will become close to diagonal, and thus it makes sense to consider an approximate estimate of $\mathbf{\Sigma}^2$ as $\hat{\mathbf{\Sigma}}^2 \approx \hat{\mathbf{D}}$. Substituting this back into (11), we obtain the final compressed log-likelihood:

$$\log p\left(\mathbf{x}_0,\ldots,\mathbf{x}_{M-1}\right) = -\log\det\left(\hat{\mathbf{R}}\right) - \sum_{i=P+1}^{L}\left[\beta_i - \log\beta_i\right], \quad (12)$$

where β_i denotes the i-th largest eigenvalue of the sample spatial coherence matrix $\hat{\mathbf{C}} \doteq \hat{\mathbf{D}}^{-1/2}\hat{\mathbf{R}}\hat{\mathbf{D}}^{-1/2}$. Then, the asymptotic log-GLRT is

$$\log\mathscr{L} \approx \sum_{i=1}^{P}\left[\log\beta_i - \beta_i\right] = \log\prod_{i=1}^{P}\beta_i e^{-\beta_i}. \quad (13)$$

That is, the test statistic is given by the product of the P largest eigenvalues of $\hat{\mathbf{C}}$, each equalized by an exponential term. Note that $\beta e^{-\beta}$ is maximum at $\beta = 1$. Thus, the statistic $\prod_{i=1}^{P}\beta_i e^{-\beta_i}$ measures how far the vector of the P largest eigenvalues $[\beta_1 \cdots \beta_P]$ is from the vector of all ones.

4. NUMERICAL RESULTS

The performance of the proposed detector is tested by means of Monte Carlo simulations. The noise level at each antenna

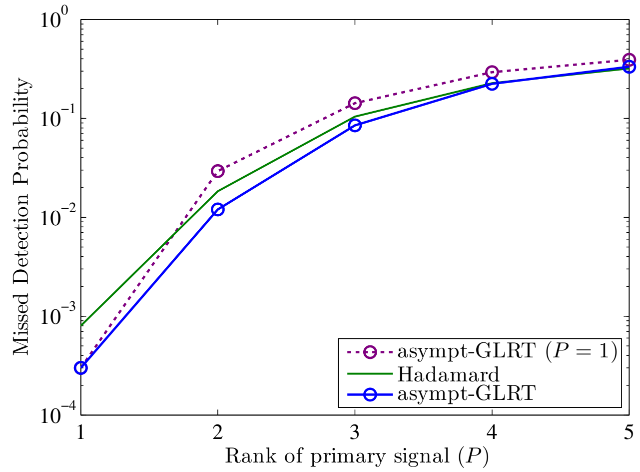

Fig. 1. Missed detection probability versus P.

is fixed during the experiment, and for each Monte Carlo realization the channel matrix $\mathbf{H}$ is generated from an uncorrelated Rayleigh distribution and scaled so that the SNR,

$$\mathrm{SNR\ (dB)} \doteq 10\log_{10}\frac{\mathrm{tr}(\mathbf{H}\mathbf{H}^H)}{\mathrm{tr}(\mathbf{\Sigma}^2)}, \quad (14)$$

remains constant during the experiment.

In the first experiment, we analyze the effect of the rank P on the performance of the proposed detector. To this end we compare the proposed asymptotic detector (13), denoted here as *asympt-GLRT*, with the GLRT previously derived in [8] and [9] for the extreme cases $P = 1$ and $P \geq L - \sqrt{L}$ (*Hadamard*), respectively. We consider a system with the following parameters: SNR $= -6$ dB, $L = 6$, $M = 128$, and the noise levels at each of the antennas are fixed to 0, -1, 1, 0.5, -1 and 0.5 dB. In Fig. 1 we show the missed detection probability (for a given fixed $P_{\mathrm{FA}} = 0.01$) of the different detection schemes against the primary signal spatial rank. First, we note that the performance of all detectors degrades for larger P. This may be explained by the fact that as P increases, the covariance matrix is losing spatial structure which, in general, helps to improve detection. We can see that the proposed asymptotic detector outperforms the other two schemes for intermediate values of P. The detector derived for $P = 1$ shows a performance loss when this assumption does not hold. On the other hand, for $P \geq 4$ the Hadamard ratio test shows similar performance to that of the rank-based detector, at a lower computational cost. This is in agreement with the result from Lemma 1.

In the second experiment, we evaluate the effect of the SNR in the performance of the proposed detector (13), which was designed for a low SNR regime. The system parameters are the same as in the previous experiment except for the SNR, which is swept between -10 dB and -3 dB. Fig. 2

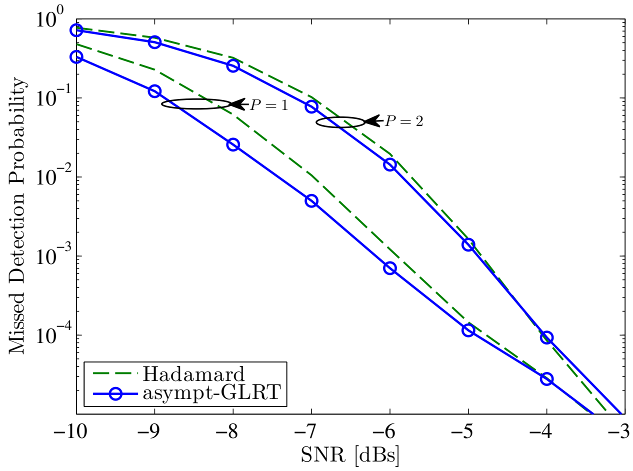

Fig. 2. Missed detection probability versus SNR for different detectors.

shows the missed detection probability of the proposed detector and that of the Hadamard ratio detector for two values of $P = 1, 2$. In Fig. 2 it is seen that whereas the asymptotic GLRT outperforms the Hadamard Ratio test for low SNR values, its performance advantage is reduced as the SNR increases. This effect can be avoided if we employ the exact GLRT obtained after numerical optimization of (11). Note that such scheme would be considerably more complex that the proposed closed-form detector.

5. CONCLUSIONS

We have presented a novel multiantenna detector for rank-P signals in spatially uncorrelated noises with different variances. In particular, we propose to use the GLRT for detecting a rank-P vector-valued random variable in non-iid noises, which generalizes several previous schemes derived either for $P = 1$ or for large P. Although this detector does not admit, in general, a close-form solution, we have shown that in the low SNR regime it reduces to a one-shot practical detector. Finally, we note that the derivation considered here assumes knowledge of the signal rank P. While this may be reasonable in some contexts, there are scenarios in which P is unknown. Future research should consider estimation of P [13] and primary signal detection jointly.

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