

DETECTION DIVERSITY OF MULTIAN TENNA SPECTRUM SENSORS

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ABSTRACT

In the context of spectrum sensing, we investigate the performance of detectors equipped with M antennas (co-located or distributed) under Rayleigh fading, in terms of *detection diversity*. Rather than the high-SNR concept of diversity order common in the communications literature, we adopt the notion recently advocated by Daher and Adve in the radar community: the slope of the average probability of detection (\bar{P}_D) vs. SNR curve at $\bar{P}_D = 0.5$. This definition is well suited to spectrum sensing, which invariably deals with low SNR levels. It is shown that the diversity order grows as M for an optimal centralized detector having access to all observations, whereas for the two distributed schemes considered (the multi-antenna energy detector and the OR detector) it grows no faster than \sqrt{M} .

Index Terms— Cognitive radio, spectrum sensing, detection diversity.

1. INTRODUCTION

Spectrum sensing is a key task in the Cognitive Radio (CR) paradigm in order to limit harmful interference to licensed users of the frequency band. By using multiple antennas (either co-located or spatially distributed), the sensing system can substantially improve its detection performance. In order to quantify and compare the performance of different multi-antenna detectors, some measure of the gain due to detection diversity is required. One could adopt a definition analogous to the one from the communications literature, i.e. the asymptotic slope of the average probability of missed detection with respect to the signal-to-noise ratio (SNR) in a log-log scale, for a fixed threshold (hence constant false alarm rate) [1] or for a SNR-dependent threshold [2]. A similar asymptotic definition based on J -divergence is given in [3].

In approaches in the communications area, the diversity order is a high-SNR concept. This is not a problem as wireless systems usually achieve this asymptotic behavior in terms of bit error rate at practical SNR values. However, spectrum sensors for CR systems are expected to provide high detec-

tion performance at much lower SNR values. This calls for a different definition of the diversity order better suited to the detection problem. For example, Daher and Adve [4] define diversity order as the slope of the average probability of detection (\bar{P}_D) curve with respect to the SNR at $\bar{P}_D = 0.5$. Using this, [4] analyzed the performance of distributed radar detectors with Swerling-II targets, in which each radar sensor is equipped with multiple antennas, the steering vectors are known and a single snapshot is used per sensor for detection. When sensing on wireless channels, these assumptions have to be suitably modified: antennas are usually omnidirectional, and sensing times are longer in order to acquire several signal samples.

In this paper, we characterize different spectrum sensing schemes in independent Rayleigh fading in terms of the Daher-Adve diversity order. Three practical schemes are considered: a Generalized Likelihood Ratio (GLR) detector, adequate for sensors with co-located antennas; and the Energy Detector (ED) and OR detector, which are amenable to distributed implementations.

2. SYSTEM MODEL

The detection system monitoring a given frequency band has M antennas, and collects K samples per antenna. The complex baseband representation of these samples is

$$\mathbf{y}_m = h_m \mathbf{x} + \sigma \mathbf{w}_m, \quad 1 \leq m \leq M \quad (K \times 1). \quad (1)$$

Here h_m is the channel coefficient from the primary transmitter to antenna m ; \mathbf{x} is the vector of primary signal samples; \mathbf{w}_m is the vector of noise samples at antenna m ; and σ^2 is the noise power, assumed known and equal across the antennas. We model \mathbf{x} and $\{\mathbf{w}_m\}$ as zero-mean white complex circular Gaussian, power normalized, jointly independent, and temporally white. In terms of $\mathbf{Y} \doteq [\mathbf{y}_1 \cdots \mathbf{y}_M]$, $\mathbf{h} \doteq [h_1 \cdots h_M]^H$ and $\mathbf{W} \doteq [\mathbf{w}_1 \cdots \mathbf{w}_M]$, we can write

$$\mathbf{Y} = \mathbf{x} \mathbf{h}^H + \sigma \mathbf{W}. \quad (2)$$

The only dependence of the pdf of \mathbf{Y} on the data is through the sample covariance matrix $\hat{\mathbf{R}} \doteq \frac{1}{K} \mathbf{Y}^H \mathbf{Y}$. The true covariance is $\mathbf{R} \doteq E\{\hat{\mathbf{R}}\} = \sigma^2 \mathbf{I} + \mathbf{h} \mathbf{h}^H$, and the pdf is then

$$f(\mathbf{Y}|\mathbf{R}) = \left[\frac{1}{\pi \det \mathbf{R}} \exp\{-\text{tr}(\mathbf{R}^{-1} \hat{\mathbf{R}})\} \right]^K. \quad (3)$$

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The corresponding hypothesis test is cast as follows:

$$\mathcal{H}_0 : \mathbf{R} = \mathbf{R}_0 \doteq \sigma^2 \mathbf{I}, \quad \mathcal{H}_1 : \mathbf{R} = \mathbf{R}_1 \doteq \sigma^2 \mathbf{I} + \mathbf{h}\mathbf{h}^H. \quad (4)$$

3. OPTIMAL AND PRACTICAL DETECTORS

The Neyman-Pearson detector (likelihood ratio test) is

$$\ell_{\text{NP}} \doteq -2 \log \frac{f(\mathbf{Y}|\mathbf{R}_0)}{f(\mathbf{Y}|\mathbf{R}_1)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \quad (5)$$

where γ is a suitable threshold. For our model,

$$\ell_{\text{NP}} \doteq -2K \log(1 + M\zeta) + \frac{2K}{\sigma^4} \frac{\mathbf{h}^H \hat{\mathbf{R}} \mathbf{h}}{(1 + M\zeta)} \quad (6)$$

with $\zeta \doteq \|\mathbf{h}\|^2/(M\sigma^2)$ the instantaneous SNR. Thus the optimal test compares $\mathbf{h}^H \hat{\mathbf{R}} \mathbf{h}$ against a threshold. As \mathbf{h} is unknown in practice, this detector is not implementable.

3.1. Generalized Likelihood Ratio Detector

The GLR detector uses the Maximum Likelihood estimate of \mathbf{h} under \mathcal{H}_1 in lieu of its true value, resulting in a test comparing the largest eigenvalue of $\hat{\mathbf{R}}$ against a threshold [5]:

$$T_{\text{GLR}} \doteq \lambda_{\max} \left(\frac{1}{\sigma^2} \hat{\mathbf{R}} \right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_{\text{GLR}}. \quad (7)$$

Under \mathcal{H}_0 the observations are temporally and spatially white, hence $K\hat{\mathbf{R}}$ is a complex Wishart matrix. The distribution of $\frac{KT_{\text{GLR}} - \mu}{\nu}$ follows asymptotically (in K and M) a Tracy-Widom distribution [6], with bias and scale given respectively by $\mu \doteq (\sqrt{K-1} + \sqrt{M})^2$ and $\nu \doteq (\sqrt{K-1} + \sqrt{M}) \left(\frac{1}{\sqrt{K-1}} + \frac{1}{\sqrt{M}} \right)^{1/3}$, that is,

$$\lambda_0 \doteq E\{T_{\text{GLR}}|\mathcal{H}_0\} = (\mu - 1.77\nu)/K. \quad (8)$$

This asymptotic pdf is accurate for moderate K , M , so γ_{GLR} can be computed for a fixed false alarm rate P_{FA} .

Under \mathcal{H}_1 , and asymptotically in K , one has [5]

$$T_{\text{GLR}} \sim \mathcal{N} \left(\lambda_1 + \frac{(M-1)\lambda_1}{K(\lambda_1-1)}, \lambda_1^2/K \right), \quad (9)$$

with $\lambda_1 \doteq 1 + M\zeta$ the largest eigenvalue of \mathbf{R}/σ^2 . Then the probability of missed detection becomes

$$P_{\text{MD}} = Q \left(\frac{\lambda_1 - \gamma_{\text{GLR}} + \frac{(M-1)\lambda_1}{K(\lambda_1-1)}}{\lambda_1/\sqrt{K}} \right). \quad (10)$$

3.2. Energy Detector

The GLR detector requires crossproducts between the received signals at each pair of antennas and hence it is not well suited for distributed networks. In these scenarios, each node could send its observed energy to a fusion center, where all such quantities are added together. This scheme is thus a multiantenna Energy Detector (ED):

$$T_{\text{ED}} = \sum_{m=1}^M \frac{\|\mathbf{y}_m\|^2}{MK\sigma^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_{\text{ED}}. \quad (11)$$

For large K , T_{ED} is approximately Gaussian so that the probabilities of false alarm and missed detection become

$$P_{\text{FA}} = Q \left((\gamma_{\text{ED}} - 1) \sqrt{MK} \right), \quad (12)$$

$$P_{\text{MD}} = Q \left(\frac{(1 + \zeta) - \gamma_{\text{ED}}}{\sqrt{(M\zeta^2 + 2\zeta + 1)/(MK)}} \right). \quad (13)$$

3.3. OR Detector

Another distributed scheme is the OR detector, in which only local decisions $u_m \in \{0, 1\}$ are sent to the fusion center, which declares \mathcal{H}_1 true if $u_m = 1$ for at least one value of m . Note that since σ^2 is known, the optimal local decision is based on the observed energy:

$$\frac{\|\mathbf{y}_m\|^2}{K\sigma^2} \underset{u_m=1}{\overset{u_m=0}{\gtrless}} \gamma_{\text{OR}}, \quad (14)$$

where we assumed equal thresholds (γ_{OR}) in the absence of any *a priori* knowledge. The global P_{FA} and P_{MD} are

$$P_{\text{FA}} = 1 - \prod_{m=1}^M (1 - P_{\text{FA}}^{(m)}), \quad P_{\text{MD}} = \prod_{m=1}^M P_{\text{MD}}^{(m)}, \quad (15)$$

with $P_{\text{FA}}^{(m)}$ and $P_{\text{MD}}^{(m)}$ respectively given by (12) and (13) when $M = 1$, $\gamma_{\text{ED}} \leftarrow \gamma_{\text{OR}}$, and $\zeta = \zeta_m \doteq |h_m|^2/\sigma^2$.

4. DIVERSITY ORDER ANALYSIS

Consider a slow fading scenario in which the channel gains remain constant during the sensing window. Assuming a fixed threshold, the probability of detection P_{D} is a random variable with expected value given by

$$\bar{P}_{\text{D}}(\bar{\zeta}) \doteq E_{\zeta}\{P_{\text{D}}\} = \int_0^{\infty} f_{\zeta}(\zeta) P_{\text{D}}(\zeta) d\zeta, \quad (16)$$

with $f_{\zeta}(\zeta)$ the pdf of ζ , and $\bar{\zeta} \doteq E_{\zeta}\{\zeta\}$ the mean SNR. Let the *minimum operational SNR* $\bar{\zeta}^*$ of the detector be defined by $\bar{P}_{\text{D}}(\bar{\zeta}^*) = 0.5$. Following [4], the diversity order d is defined as

$$d \doteq \left. \frac{\partial \bar{P}_{\text{D}}(\bar{\zeta})}{\partial \bar{\zeta}} \right|_{\bar{\zeta}=\bar{\zeta}^*}, \quad \text{with } \bar{P}_{\text{D}}(\bar{\zeta}^*) = \frac{1}{2}. \quad (17)$$

We assume uncorrelated Rayleigh fading, i.e. \mathbf{h} is zero-mean circular complex Gaussian with covariance $E\{\mathbf{h}\mathbf{h}^H\} = \rho^2 \mathbf{I}$. Then $\bar{\zeta} = \rho^2/\sigma^2$, and ζ has the following pdf [7]:

$$f_{\zeta}(\zeta) = \frac{M^M}{(M-1)!} \frac{\zeta^{M-1}}{\bar{\zeta}^M} \exp\{-M\zeta/\bar{\zeta}\}, \quad \zeta > 0. \quad (18)$$

Unfortunately, (16) does not admit a closed form solution for any of the detectors discussed in Sec. 3. We propose the following first-order piecewise approximation of $P_{\text{MD}}(\zeta)$, where ζ^* is such that $P_{\text{MD}}(\zeta^*) = 0.5$:

$$P_{\text{MD}}(\zeta) \approx \begin{cases} 1, & 0 < \zeta < \zeta_1, \\ \frac{1}{2} - a(\zeta - \zeta^*), & \zeta_1 < \zeta < \zeta_2, \\ 0, & \zeta > \zeta_2, \end{cases} \quad (19)$$

where $\zeta_1 \doteq \zeta^* - \frac{1}{2a}$, $\zeta_2 \doteq \zeta^* + \frac{1}{2a}$ and

$$a \doteq - \left. \frac{\partial P_{\text{MD}}(\zeta)}{\partial \zeta} \right|_{\zeta=\zeta^*} = \left. \frac{\partial P_{\text{D}}(\zeta)}{\partial \zeta} \right|_{\zeta=\zeta^*}. \quad (20)$$

Substituting now (18) and (19) into (16) one obtains

$$\bar{P}_{\text{MD}} \approx a \left\{ \zeta_2 \Gamma\left(\frac{M\zeta_2}{\bar{\zeta}}, M\right) - \zeta_1 \Gamma\left(\frac{M\zeta_1}{\bar{\zeta}}, M\right) - \bar{\zeta} \left[\Gamma\left(\frac{M\zeta_2}{\bar{\zeta}}, M+1\right) - \Gamma\left(\frac{M\zeta_1}{\bar{\zeta}}, M+1\right) \right] \right\} \quad (21)$$

where the incomplete Gamma function is defined as

$$\Gamma(x, \alpha) \doteq \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-t} dt, \quad (22)$$

with $\Gamma(\alpha) \doteq \int_0^{\infty} t^{\alpha-1} e^{-t} dt$ the standard Gamma function.

Taking derivatives in (21), noting that $\bar{P}_{\text{D}} = 1 - \bar{P}_{\text{MD}}$, and after some algebra, one arrives at

$$d \approx a \left[g_M\left(\frac{\zeta^*}{\bar{\zeta}} + \frac{1}{2a\bar{\zeta}^*}\right) - g_M\left(\frac{\zeta^*}{\bar{\zeta}} - \frac{1}{2a\bar{\zeta}^*}\right) \right], \quad (23)$$

where $g_M(x) \doteq \Gamma(Mx, M+1)$. While (23) may look like a *rough* approximation of the diversity order, we will show in Sec. 5 that it effectively captures the behavior of \bar{P}_{D} in Rayleigh fading environments.

4.1. GLR detector performance

Using the asymptotic distribution (10), one readily obtains the parameters ζ^* and a for this detector:

$$\zeta_{\text{GLR}}^* = \frac{1}{2M} \left[\beta + \sqrt{(2+\beta)^2 - 4\gamma_{\text{GLR}}} \right], \quad (24)$$

$$a_{\text{GLR}} = \sqrt{\frac{KM^2}{2\pi}} \frac{1 - \frac{M-1}{K(M\zeta_{\text{GLR}}^*)^2}}{1 + M\zeta_{\text{GLR}}^*}, \quad (25)$$

with $\beta \doteq \gamma_{\text{GLR}} - \frac{K+M-1}{K}$.

Now, finding the value of $\bar{\zeta}^*$ at which (21) equals 0.5 is not straightforward. However, an obvious candidate is $\bar{\zeta}^* \approx$

ζ_{GLR}^* , since the *instantaneous* probability of missed detection satisfies $P_{\text{MD}}(\zeta_{\text{GLR}}^*) = 0.5$. With $\epsilon_{\text{GLR}} \doteq \frac{1}{2a_{\text{GLR}}\zeta_{\text{GLR}}^*}$, this yields

$$d_{\text{GLR}} \approx a_{\text{GLR}} [g_M(1 + \epsilon_{\text{GLR}}) - g_M(1 - \epsilon_{\text{GLR}})], \quad (26)$$

where both a_{GLR} and ζ_{GLR}^* depend on the system parameters K , M and P_{FA} . Noting that the bracketed term in (26) is less than 1, the following upper bound is obtained:

$$d_{\text{GLR}} < a_{\text{GLR}} < \sqrt{\frac{KM^2}{2\pi}}. \quad (27)$$

It can be shown that as $M \rightarrow \infty$ and for $\epsilon > 0$, $g_M(1+\epsilon) \rightarrow 1$ whereas $g_M(1-\epsilon) \rightarrow 0$. Thus, for large M , $d_{\text{GLR}} \approx a_{\text{GLR}}$.

4.2. Energy Detection performance

In this case, the parameters for the first-order piecewise approximation of (13) are $\zeta_{\text{ED}}^* = \gamma_{\text{ED}} - 1$ and

$$a_{\text{ED}} = \sqrt{\frac{KM}{2\pi}} \frac{1}{\sqrt{M(\zeta_{\text{ED}}^*)^2 + 2\zeta_{\text{ED}}^* + 1}}, \quad (28)$$

so that

$$d_{\text{ED}} < a_{\text{ED}} < \sqrt{\frac{KM}{2\pi}}, \quad (29)$$

with $d_{\text{ED}} \rightarrow a_{\text{ED}}$ as $M \rightarrow \infty$.

4.3. OR detection performance

Defining the vector of local SNRs $\zeta \doteq [\zeta_1 \cdots \zeta_M]^T$, the probability of missed detection of the OR detector is

$$\bar{P}_{\text{MD}} = \int f_{\zeta}(\zeta) P_{\text{MD}}(\zeta) d\zeta = \left[\int_0^{\infty} f_{\zeta}(\zeta) P_{\text{MD}}^{(m)}(\zeta) d\zeta \right]^M.$$

The same technique as in the previous sections, particularized for $M = 1$, can be used now to approximate the integral. After some algebra, one arrives at

$$\bar{P}_{\text{MD}}(\bar{\zeta}) \approx \left[1 - 2a_{\text{OR}}\bar{\zeta} \sinh\left(\frac{1}{2a_{\text{OR}}\bar{\zeta}}\right) e^{-\zeta_{\text{OR}}^*/\bar{\zeta}} \right]^M, \quad (30)$$

$$\zeta_{\text{OR}}^* = \frac{Q^{-1}(1 - \sqrt[M]{1 - P_{\text{FA}}})}{\sqrt{K}}, \quad a_{\text{OR}} = \sqrt{\frac{K}{2\pi}} \frac{1}{1 + \zeta_{\text{OR}}^*}.$$

Taking the derivative of (30), one finds that

$$d_{\text{OR}} \approx \frac{M(\sqrt[M]{2} - 1)}{2a_{\text{OR}}\bar{\zeta}^*} \left[\frac{\zeta_{\text{OR}}^* - \frac{1}{2a_{\text{OR}}}}{\bar{\zeta}^*} + 1 \right]. \quad (31)$$

One must solve for $\bar{\zeta}^*$ in $\bar{P}_{\text{MD}}(\bar{\zeta}^*) = \frac{1}{2}$ in (30), i.e.,

$$1 - \frac{1}{\sqrt[M]{2}} = 2a_{\text{OR}}\bar{\zeta}^* \sinh\left(\frac{1}{2a_{\text{OR}}\bar{\zeta}^*}\right) e^{-\zeta_{\text{OR}}^*/\bar{\zeta}^*}, \quad (32)$$

which can be easily solved numerically.

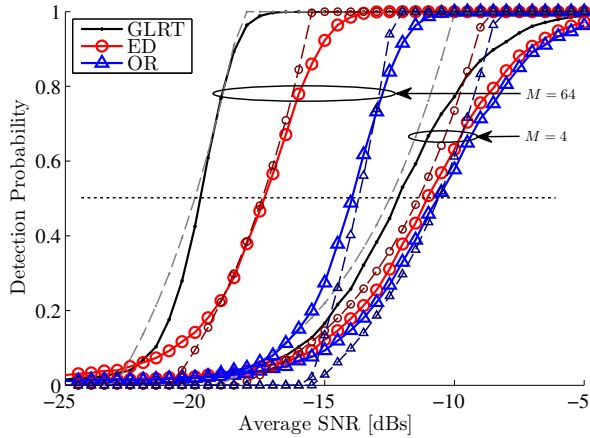


Fig. 1. Accuracy of the proposed approximations. Solid lines: simulation results. Dashed lines: analytical approximations. $P_{FA} = 0.01$, $K = 256$.

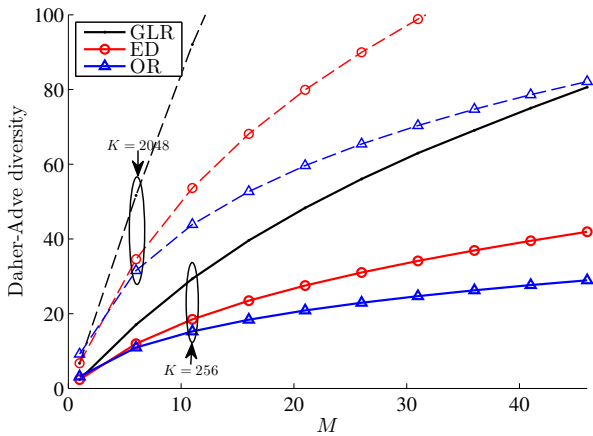


Fig. 2. Diversity order d of the three detectors.

5. NUMERICAL RESULTS

We fix $P_{FA} = 0.01$ throughout. First we check the accuracy of our approximations: Fig. 1 shows the $\bar{P}_D(\bar{\zeta})$ curves for the three detectors in Rayleigh fading, obtained by Monte Carlo simulation, as well as the corresponding piecewise linear approximations from the previous sections. These match the empirical curves reasonably well around $\bar{P}_D \approx \frac{1}{2}$. Hence, the detection performance can be accurately described using just two parameters: the minimum operational SNR $\bar{\zeta}^*$ and the diversity order d .

Fig. 2 shows the analytical approximations for the diversity order as a function of M . These curves have to be considered together with those in Fig. 3 for the minimum operational SNR $\bar{\zeta}^*$. From Figs. 2 and 3 the performance advantage of the GLR detector is clear. The diversity order of this centralized detector grows almost linearly with the number of antennas, whereas that of the ED is approximately propor-

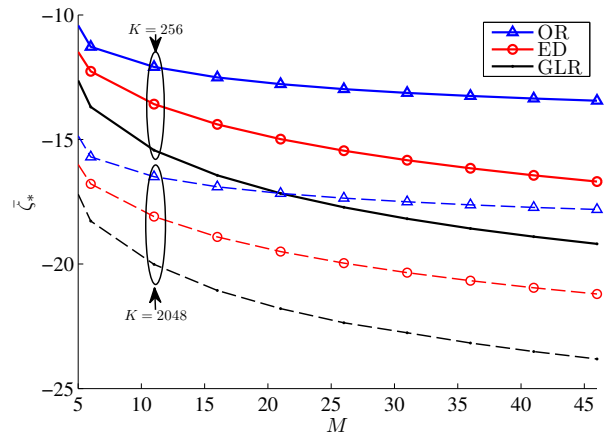


Fig. 3. Minimum operational SNR of the three detectors.

tional to \sqrt{M} . As for the OR detector, it is difficult to find analytical bounds for its diversity order in terms of M . By comparison with ED, it is seen in Fig. 2 that it increases at a rate no larger than \sqrt{M} . It is conjectured that the diversity order of the OR detector is logarithmic in M , similarly to [4].

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